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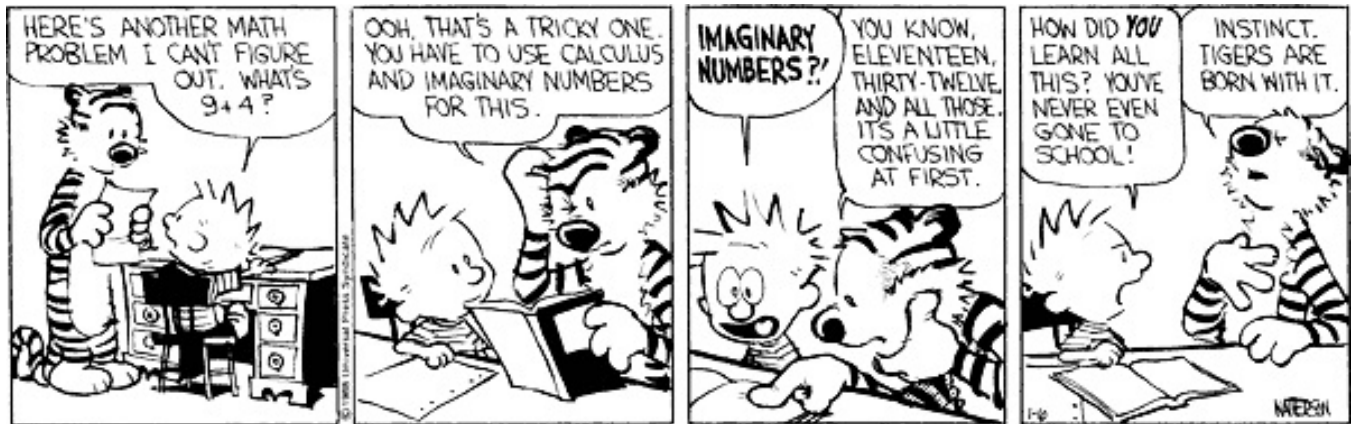
Date: _____

Do not start this exam until instructed; you will have 90 minutes to finish the exam. No notes, books, calculators, phones or electronic devices are allowed on this exam. If you have a question, raise your hand; otherwise, there is no talking during the exam.

Show all your work to receive full credit.

There are . problems on this exam on . pages, in addition to this cover page. The point values of each problem vary, but are listed in the questions.

Good luck!



From *Calvin and Hobbes*.

1. (5+5+5=15 points) Set up, but **do not evaluate**, integrals to represent the following. You may use whatever coordinate system is convenient.

(a) The volume of the region where

$$1 \leq x^2 + y^2 + z^2 \leq 4, \quad z \geq 0$$

(b) The volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 10$ and $x + z = -5$.

(c) The volume above the rectangle $[1, 3] \times [1, 4]$ and below the cylinder $z = e^x$.

2. (4+4+4+3=15 points) (a) Write a triple integral to represent the volume above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = 9$ using

(i) Rectangular coordinates

(ii) Cylindrical coordinates

(iii) Spherical coordinates.

(b) Calculate the volume using one of the expressions from part (a).

3. (4 points) Evaluate the following iterated integral:

$$\int_0^1 \int_0^2 xy + y \, dy \, dx$$

4. (8 points) Sketch the region described by $D = \{(x, y) : 0 \leq x \leq 1, x \leq y \leq 1\}$. Using an appropriate order of integration, compute

$$\iint_D \cos y^2 \, dA$$

5. (5+5=10 points) Consider the iterated integral

$$\int_0^1 \int_y^1 \int_0^y f(x, y, z) dz dx dy$$

Write iterated integrals with the following orders to represent this integral:

(a)

$$\int \int \int f(x, y, z) dz dy dx$$

(b)

$$\int \int \int f(x, y, z) dx dy dz$$

6. (5 points) The upper half of a hemisphere of radius 3 has density equal to the distance from the bottom edge. Write an iterated integral to represent the mass of this object.

7. (8 points) Let C be the curve described by $y = x^3$ from the origin $(0, 0)$ to the point $(2, 8)$; let $f(x, y) = xy^2$. Evaluate the line integral

$$\int_C f(x, y) ds$$

8. (8 points) A force field is given by $\vec{F}(x, y) = x\vec{i} + y\vec{j}$. Find the work done by this field in moving a particle from $(1, 0)$ to $(-1, 0)$ counterclockwise along the circle $x^2 + y^2 = 1$.

9. (15 points) Consider the vector field

$$\vec{F}(x, y) = \langle e^{x+y}, e^{x+y} + 2y \rangle$$

(a) Determine whether the field is conservative or not.

(b) If the field is conservative, find a function f so that $\nabla f = F$.

(c) Find the work done moving a particle from the origin to the point $(2, 4)$ along the parabola $y = x^2$.

10. (7+5=12 points) Let C be the boundary of the circle $x^2 + y^2 = 4$, oriented counterclockwise.

(a) Evaluate the line integral

$$\int_C y \, dx - x \, dy$$

directly.

(b) Evaluate the above line integral by using Green's Theorem.

Extra Credit. This problem is substantially more difficult - only attempt it if you have finished the rest of the exam. Your score can be increased by up to 10 points, but not above 100 total.

11. (10 points) Find the positively oriented simple closed curve C for which the value of the line integral

$$\int_C (y^3 - y) dx - 2x^3 dy$$

is maximized.